# The Impact Parameters of the Broadening and Shift of Spectral Lines 

M. S. Helmi and G. D. Roston*<br>Department of Physics, Faculty of Science, Alexandria University, Alexandria, Egypt

(received February 25, 2003; revised January 19, 2005; accepted January 25, 2005)


#### Abstract

The classical theory of collisional broadening and shift coefficients $(\beta, \delta)$ of an isolated spectral line was used to obtain simple analytical formulae for calculating both $\beta$ and $\delta$. These formulae were obtained on the assumption that the short range interaction was effective only in the broadening, while the long range was effective in the shift of the spectral line. These coefficients, $\beta$ and $\delta$ depended on the limiting phase shifts responsible for broadening $\eta_{0 \mathrm{~b}}$ and shift $\eta_{08}$. It was found that the values of $\eta_{0 \mathrm{~b}}$ and $\eta_{0 \delta}$ were not equal to each other, as was proposed by Weisskopf $\eta_{0 \mathrm{~b}}=\eta_{0 \delta}=1$, but were instead $\eta_{\mathrm{ob}}= \pm \pi / 5$ and $\eta_{\mathrm{o} \delta}= \pm \pi / 2$. The correct signs of these phases were obtained and defined. When these phases were applied with their correct signs in the approximate formulae, the broadening $\beta_{c}$ and shift $\delta_{c}$ coefficients for some interactions of $\mathrm{Tl}, \mathrm{Hg}, \mathrm{Cd}, \mathrm{Zn}, \mathrm{Ar}$ and Ne with inert gases and self-interactions were in agreement with the corresponding values obtained numerically by other authors. The limit at which the shift changed its sign was also obtained. New impact parameters which were not known up to now have been discussed and obtained.


Keywords: spectral line, broadening coefficient $\beta$, shift coefficient $\delta$, impact parameters, spectral line shift, spectral line broadening

## Introduction

It has long been appreciated that studying the collision broadening and shift of spectral lines contain information concerning the interatomic potentials between the radiating and perturbing atoms. The theoretical treatment of this process is of great interest for the region of low densities at which the interactions of two particles are predominant, where the impact approximation takes place, and the half-width and shift are proportional to the density of the perturbing gas. In this case, the broadening and shift coefficients $\beta$ and $\delta$, respectively, are specified for such interactions. Quantum mechanical formulation of the impact approximation to the theory of collision broadening and shift of spectral lines gives results which differ little from those of the classical theory. It seems worthwhile, therefore, to use the classical theory of collision broadening and shift developed for any interaction potential. To interpret the experimental data, the theoretical values of broadening and shift parameters in the impact limit of line broadening theory have been calculated for van der Waals and Lennard-Jones potentials by many authors (Dygdala et al., 1989; Dygdala, 1988; Bielski et al., 1985; Czychaj and Sienkiewicz, 1984). The results of these calculations were obtained by the numerical solution of the Lindholm and Foley impact theory of broadening developed by Hindmarsh et al. (1967) and Helmi (1994), who concluded that the impact parameters for the broadening and shift coefficients of spectral lines must be different, contrary to the proposed value $\rho_{0 \beta}=$

[^0]$\rho_{0 \delta}=\rho_{0}$ due to the Weisskopf phase shift $\eta_{0}=1$ (Helmi, 1994), which has no basis. Helmi and Roston (2000) obtained simple analytical formulae for calculating $\beta$ and $\delta$ in case of LennardJones potential. These formulae were based on the assumption that the phase shifts for the broadening $\eta_{\mathrm{ob}}$ and shift $\eta_{\mathrm{od}}$ were different. These values are given by $\eta_{o \mathrm{ob}}= \pm \pi / 5$ and $\eta_{o \delta}$ $= \pm \pi / 2$. Comparing the calculated values of $\beta$ and $\delta$ with that calculated by the numerical method (Hindmarsh et al., 1967), it was found that there was a good agreement between the two values for some interactions, when $\eta_{o b}$ and $\eta_{o \delta}$ were positive and when other interactions were negative.

The aim of this work was to obtain the correct sign of the broadening and shift phases $\eta_{\mathrm{ob}}$ and $\eta_{o \delta}$ and how to apply these to calculate the values of the new impact parameters, $\rho_{\mathrm{ob}}$ and $\rho_{\mathrm{od} \delta}$, responsible for the broadening and shift of spectral line, and also to obtain the critical value of the impact parameter, $\rho_{\delta}$, which separates between the red and blue shifts of the spectral line.

## Theoretical Background

According to the adiabatic phase shift theory, the broadening $\beta$ and shift $\delta$ coefficients are given by:
$\beta=4 \pi \hat{v} \int_{0}^{\infty} \rho[1-\cos \eta(\hat{v}, \rho)] d \rho$
$\delta=2 \pi \hat{v} \int_{0}^{\infty} \rho \sin \eta(\hat{v}, \rho) d \rho$
where:
$\eta(\hat{v}, \rho)=$ the total phase shift caused by a single collision occurring at the impact parameter $\rho$ and relative velocity $\hat{v}$.

If the perturber follows a straight line trajectory, $\eta(\hat{v}, \rho)$ can be written as (Findeisen et al., 1987):
$\eta(\hat{v}, \rho)=\frac{2}{\hbar \hat{v}} \int_{0}^{\infty} \frac{R \Delta V(R)}{\left[R^{2}-\rho^{2}\right]^{1 / 2}} d R$
where:
$R=$ the interatomic separation
$\Delta \mathrm{V}(\mathrm{R})=$ the difference between the adiabatic potentials describing the interaction between the perturber and emitting atom in its upper and lower states

For the simplest case of monatomic inverse-power potentials $\Delta \mathrm{V}(\mathrm{R})=\hbar \mathrm{C}_{\mathrm{n}} \mathrm{R}^{-\mathrm{n}}$, the phase shift $(\hat{v}, \rho)$ takes the form:
$\eta(\hat{v}, \rho)=\frac{\alpha_{n} C_{n}}{\hat{v} \rho^{n-1}}$
where:
$\alpha_{n}=\sqrt{\pi} \frac{\Gamma(n-1) / 2}{\Gamma(n / 2)}$
The approximate formulae for $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$. If we assume that the effective part for broadening $\beta$ in (1a) comes from the near distances ( $0-\rho_{0 b}$ ) and the effective part for the shift $\delta$ in (1b) comes from the far distances ( $\rho_{0 \delta} \rightarrow \infty$ ), then (1a) and (1b) will take the form:
$\beta=4 \pi \hat{v} \int_{0}^{\rho_{o b}} \rho[1-\cos \eta(\hat{v}, \rho)] d \rho$
$\delta=2 \pi \hat{v} \int_{\rho_{o \delta}}^{\infty} \rho \sin \eta(\hat{v}, \rho) d \rho$
It may be seen from (3) that $\eta(\hat{v}, \rho)$ will be large in the short range ( $0-\rho_{o b}$ ) and $\cos \eta(\hat{v}, \rho)$ is quickly oscillating, so that $\int_{\cos } \eta(\hat{v}, \rho)=0$, while $\eta(\hat{v}, \rho)$ will be very small in the long range $\left(\rho_{0 \delta} \rightarrow \infty\right)$, so that $\sin \eta(\hat{v}, \rho) \approx \eta(\hat{v}, \rho)$. From this was found that (Helmi and Roston, 2000):
$\beta=2 \pi \hat{v} \int_{0}^{\rho_{o b}} \rho d \rho=2 \pi \rho_{0 \mathrm{~b}}^{2}$
$\delta=2 \pi \hat{v} \int_{\rho_{0 \delta}}^{\infty} \rho \eta(\hat{v}, \rho) d \rho$
where:
$\rho_{0 \mathrm{~b}}$ and $\rho_{0 \delta}=$ respectively, the broadening and shift impact parameters

Applications to potentials. Different types of inverse-power potentials. The approximate formulae (6) are used to express $\beta$ and $\delta$ in terms of the limiting values of impact parameter $\rho$, and hence, via (3), the limiting values of $\eta$. Introducing the van der Waal's difference potential $\Delta \mathrm{V}(\mathrm{R})=h \mathrm{C}_{\mathrm{n}} \mathrm{R}^{-\mathrm{n}}$ in (2), the phase
parameters due to the broadening $\eta_{0 \mathrm{~b}}$ and shift $\eta_{0 \delta}$ are given as (Helmi and Roston, 2000):
$\eta_{\mathrm{ob}}=\left[2 \int_{0}^{\infty} x\left[1-\cos x^{1-\mathrm{n}}\right] d x\right]^{\frac{1-n}{2}}$
$\eta_{08}=\left[(n-3) \int_{0}^{\infty} x \sin x^{1-n} d x\right]^{\frac{1-n}{3-n}}$
The average values of $\eta_{0 \mathrm{~b}}$ and $\eta_{0 \delta}$ for diffrent values of $\mathrm{n}[\mathrm{n}=$ 3, 4 and 6] were calculated by Helmi and Roston (2000), which gives $\eta_{\mathrm{ob}}= \pm \pi / 5$ and $\eta_{\mathrm{o} \mathrm{\delta}}= \pm \pi / 2$.

Lennard-Jones potential. The Lennard-Jones difference potential between the excited and the ground states of the quasimolecule consisting of radiating and perturbing atoms is given by:
$\Delta V(R)=\hbar \Delta C_{12} R^{-12}-\hbar \Delta C_{6} R^{-6}$
where:
$\mathrm{R}=$ the distance between the colliding atoms
$\Delta \mathrm{C}_{6}, \Delta \mathrm{C}_{12}=$ constants, depending on the states of these atoms

The pressure broadening $\beta$ and shift $\delta$ coefficients in the case of L-J potential are given as (Hindmarsh et al., 1967):
$\beta=8 \pi\left(\frac{3 \pi}{8}\right)^{\frac{2}{5}} \hat{v}^{\frac{3}{5}}\left(\Delta C_{6}\right)^{\frac{2}{5}} \mathrm{~B}(\alpha)$
$\delta=2 \pi\left(\frac{3 \pi}{8}\right)^{\frac{2}{5}} \hat{v}^{\frac{3}{5}}\left(\Delta C_{6}\right)^{\frac{2}{5}} S(\alpha)$
where:
the broadening and shift functions $B(\alpha)$ and $S(\alpha)$ are defined by the following integrals:
$\mathrm{B}(\alpha)=\int_{0}^{\infty} x \sin ^{2} \frac{1}{2}\left(\alpha x^{-11}-x^{-5}\right) d x$
$S(\alpha)=\int_{0}^{\infty} x \sin \left(\alpha x^{-11}-x^{-5}\right) d x$
where:

$$
\begin{equation*}
\alpha=0.536 \hat{v}^{\frac{6}{5}}\left(\Delta C_{6}\right)^{-\frac{11}{5}} \Delta C_{12} \tag{10b}
\end{equation*}
$$

The functions $B(\alpha)$ and $S(\alpha)$ were obtained numerically for some chosen values of $\alpha$. The phase shift $\eta(\hat{v}, \rho)$ in case of L-J potential is given by introducing (8) into (2) as:
$\eta(\hat{v}, \rho)=\left(\frac{\alpha_{12} \Delta C_{12}}{\hat{v}}\right) \rho^{-11}-\left(\frac{\alpha_{6} \Delta C_{6}}{\hat{v}}\right) \rho^{-5}$
Introducing the obtained average values $\eta_{0 \mathrm{~b}}= \pm 0.63$ and $\eta_{00}=$ $\pm 1.57$ into (11), the broadening and shift impact parameters $\rho_{0 \text { b }}$ and $\rho_{0 \delta}$ in the case of L-J potential are obtained as:
$\left(\frac{\alpha_{12} \Delta C_{12}}{\hat{v}}\right) \rho_{o b}^{-11}-\left(\frac{\alpha_{6} \Delta C_{6}}{\hat{v}}\right) \rho_{o b}^{-5}= \pm \frac{\pi}{5}= \pm 0.63$

$$
\begin{equation*}
\left(\frac{\alpha_{12} \Delta C_{12}}{\hat{v}}\right) \rho_{0 \delta}^{-11}-\left(\frac{\alpha_{6} \Delta C_{6}}{\hat{v}}\right) \rho_{0 \delta}^{-5}= \pm \frac{\pi}{2}= \pm 1.57 \tag{12b}
\end{equation*}
$$

The positive sign in (12a, b) denotes that the repulsion forces are more than the attraction forces, while the negative sign denotes that the attraction forces are more than the repulsive forces at the collision time.

To obtain the broadening $\beta$ and shift $\delta$ coefficients in case of L-J potential, we introduce (11) into (6a, b), which gives:
$\beta=\hat{v} \rho_{0 \mathrm{~b}}^{2}$
$\delta=\hat{v}\left(\frac{\alpha_{12} \Delta C_{12}}{9 \hat{v}}\right) \rho_{0 \delta}^{-9}-\left(\frac{\alpha_{6} \Delta C_{6}}{3 \hat{v}}\right) \rho_{0 \delta}^{-3}$

Knowing $\rho_{0 \mathrm{~b}}$ and $\rho_{0 \delta}$ from (12a, b), $\beta$ and $\delta$ can be obtained directly from the approximated formulae (13a, b).

Applying the Lennard-Jones (L-J) potential with the same obtained average values of $\eta_{\text {оь }}$ and $\eta_{0 \delta}$, and comparing the results obtained by the approximated formulae (6a, b) for $\beta$ and $\delta$ with the results obtained for these coefficients by numerical calculations of (Hindmarsh et al., 1967), Helmi and Roston (2000) found that there was an agreement between the two results for some interactions when $\eta_{0 \mathrm{~b}}$ and $\eta_{0 \delta}$ were positive, and other interactions were negative, but Helmi and Roston (2000) did not clarify the reason of the different signs, and as to when these signs could be applied.

To obtain the necessary sign used in (12a, b), the following was proceeded:

Let $\rho_{0}, \rho_{\mathrm{E}}, \rho_{0 \mathrm{~b}}$ and $\rho_{0 \delta}$ be the impact parameters corresponding, respectively, to the phase shifts $\eta=0, \eta=\eta_{\mathrm{E}}$ (the phase shift well-depth), $\eta_{0 b}= \pm 0.63$ and $\eta_{0 \delta}= \pm 1.57$. When $\eta(\hat{v}, \rho)$ given by (11) was plotted against $\rho$ for different interactions, the plotted curves had the form shown in Fig. 1-3. The parameters $\rho_{o}, \rho_{E}$ and $\eta_{E}$ are given from (11) as:
$\rho_{0}=\left[\frac{21 \Delta C_{12}}{32 \Delta C_{6}}\right]^{1 / 6}, \quad \rho_{E}=1.14 \rho_{0}$
$\eta_{E}=\left(\frac{\alpha_{12} \Delta C_{12}}{\bar{V}}\right) \rho_{E}^{-11}-\left(\frac{\alpha_{6} \Delta C_{6}}{\bar{V}}\right) \rho_{E}^{-5}$
It may also be seen from (13b) that the impact parameter $\rho_{\delta}$, which separates between the negative and positive signs of the shift coefficients $\delta$, is given by:
$\rho_{\delta}=\left[\frac{7 \Delta C_{12}}{32 \Delta C_{6}}\right]^{1 / 6}=1.2 \rho_{0}$
So that if $\rho_{0 \delta}<\rho_{\delta}$ in (12b), then $\delta$ has a positive sign and the spectral line will be shifted to the blue wing, but if $\rho_{0 \delta}>\rho_{\delta}$,
then $\delta$ has a negative sign and the spectral line will be shifted to the red wing.

## Results and Discussion

To obtain the appropriate sign of $\eta_{0 \mathrm{~b}}= \pm 0.63$ and $\eta_{0 \delta}= \pm 1.57$, we proceed as follows:
knowing, $\Delta \mathrm{C}_{6}, \Delta \mathrm{C}_{12}$ and $\hat{v}, \eta_{\mathrm{E}}$ can be obtained using (14);
(1) if $\eta_{\mathrm{E}}>-0.63$
then:
$\rho_{0 \mathrm{~b}}$ and $\rho_{0 \delta}$ can be obtained using the positive sign for $\eta_{0 \mathrm{~b}}$ and $\eta_{0 \delta}$ according to (12a, b), in this case $\rho_{0 \delta}<\rho_{0 \mathrm{~b}}<\rho_{\mathrm{E}}$ (Fig. 1),
(2) if $\eta_{\mathrm{E}} \leq-1.57$
then:
$\rho_{0 \mathrm{~b}}$ and $\rho_{0 \delta}$ will be taken with the negative sign for $\eta_{0 \mathrm{~b}}$ and $\eta_{0 \delta}$, in this case $\rho_{0 \mathrm{~b}}>\rho_{0 \delta}>\rho_{\mathrm{E}}$ (Fig. 2),
(3) if $\eta_{E}<-0.63$, but $>-1.57$
then:
$\rho_{0 \mathrm{~b}}$ will be taken with the negative sign for $\eta_{0 \mathrm{~b}}$, so that, $\rho_{0 \mathrm{~b}}>\rho_{\mathrm{E}}$, however, $\rho_{0 \delta}$ will be taken with the positive sign for $\eta_{0 \delta}$, so that $\rho_{0 \delta}<\rho_{\mathrm{E}}$ (Fig. 3).

The calculated coefficients $\beta_{c}$ and $\delta_{c}$ for different interactions are illustrated in Table 1, with the corresponding Hindmarsh values $\beta_{\mathrm{H}}$ and $\delta_{\mathrm{H}}$, for the interactions of $\mathrm{Ar}, \mathrm{Ne}, \mathrm{Tl}, \mathrm{Hg}, \mathrm{Cd}$ and Zn atoms with the inert gases $\mathrm{Xe}, \mathrm{Kr}, \mathrm{Ar}, \mathrm{Ne}$ and He .


Fig. 1. The phase shift $\eta$ as a function of the impact parameter $\rho$, when $\eta_{E}>-0.63$.

Table 1. The calculated values of pressure broadening $\beta_{c}$ and shift $\delta_{c}$ coefficients with the corresponding Hindmarsh values $\beta_{\mathrm{H}}$ and $\delta_{\mathrm{H}}$ in units $10^{-20} / \mathrm{cm} /$ atom $/ \mathrm{cm}^{3}$ for $\mathrm{Ar}, \mathrm{Ne}, \mathrm{Tl}, \mathrm{Hg}, \mathrm{Cd}$ and Zn perturbed by inert gases*

| Perturber | Parameters |  | $\begin{gathered} \text { Phase } \\ \hline \eta_{\mathrm{E}} \end{gathered}$ | Impact parameters |  |  |  | Broadening |  | Shift potential |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta \mathrm{C}_{6}$ | $\Delta \mathrm{C}_{12}$ |  | $\rho_{\text {¢ }}$ | $\rho_{\text {E }}$ | $\rho_{\text {ob }}$ | $\rho_{\text {об }}$ | $\beta_{\text {H }}$ | $\beta_{\text {c }}$ | $\delta_{\text {H }}$ | $\delta_{C}$ |
| Ar line $703 \mathrm{~nm}, \mathrm{~T}=330 \mathrm{~K}$ |  |  |  |  |  |  |  |  |  |  |  |
| $\operatorname{Ar}[1]$ | 531.8 | 83750 | -0.060 | 18.0 | 24.7 | 18.8 | 17.6 | 7.040 | 6.930 | 0.200 | 0.200 |
| Ne [1] | 130.0 | 55870 | -0.005 | 21.3 | 29.2 | 18.4 | 17.0 | 8.250 | 8.170 | 0.990 | 1.000 |
| $\mathrm{He}[1]$ | 70.50 | 48190 | -0.001 | 23.0 | 31.7 | 17.3 | 16.0 | 13.89 | 13.75 | 1.890 | 1.780 |
| Ne line $540 \mathrm{~nm}, \mathrm{~T}=330 \mathrm{~K}$ |  |  |  |  |  |  |  |  |  |  |  |
| Ne [4] | 5.400 | 1.2400 | -0.100 | 6.09 | 8.34 | 6.52 | 6.15 | 1.153 | 1.170 | -0.028 | -0.028 |
| He[4] | 2.900 | 0.8800 | $-0.025$ | 6.37 | 8.73 | 6.22 | 5.79 | 1.870 | 1.870 | 0.150 | 0.150 |
| Tl line $377.68 \mathrm{~nm}, \mathrm{~T}=\mathbf{8 6 0} \mathrm{K}$ |  |  |  |  |  |  |  |  |  |  |  |
| Xe[2,3] | 145.2 | 30.910 | -5.290 | 5.99 | 8.21 | 14.1 | 11.7 | 3.270 | 3.160 | -1.100 | -1.170 |
| Kr[2,3]] | 49.40 | 20.610 | -1.900 | 6.08 | 9.49 | 12.4 | 10.0 | 2.840 | 2.840 | -1.400 | -1.100 |
| Ar [2,3] | 58.70 | 15.250 | -1.170 | 6.20 | 8.49 | 7.72 | 7.12 | 1.900 | 1.949 | -1.180 | -1.200 |
| $\mathrm{Ne}[2,3]$ | 14.30 | 6.5800 | -0.130 | 6.82 | 9.34 | 7.42 | 7.00 | 1.640 | 1.800 | -0.080 | -0.090 |
| Hg line $253.6 \mathrm{~nm}, \mathrm{~T}=860 \mathrm{~K}$ |  |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{Hg}[5]$ | 11.700 | 0.0572 | -10.98 | 3.19 | 4.38 | 8.76 | 7.29 | 1.079 | 1.085 | -0.396 | $-0.394$ |
| Xe[6] | 0.2970 | 0.0010 | -0.270 | 3.01 | 4.28 | 3.52 | 3.36 | 0.207 | 0.202 | -0.037 | -0.036 |
| Kr[7] | 0.7920 | 0.0080 | $-0.300$ | 3.61 | 4.94 | 4.08 | 3.93 | 0.304 | 0.314 | -0.068 | -0.067 |
| Ar[8] | 0.0630 | 0.4970 | -0.003 | 5.09 | 6.97 | 4.20 | 3.87 | 0.438 | 0.443 | 0.055 | 0.059 |
| $\mathrm{Ne}[8]$ | 0.7450 | 0.0640 | $-0.027$ | 5.16 | 7.05 | 5.07 | 4.72 | 0.850 | 0.847 | 0.064 | 0.066 |
| Cdline $326.1 \mathrm{~nm}, \mathrm{~T}=860 \mathrm{~K}$ |  |  |  |  |  |  |  |  |  |  |  |
| Cd[9] | 7.2200 | 0.0451 | -4.080 | 3.33 | 4.56 | 7.45 | 6.15 | 1.140 | 1.070 | -0.365 | -0.396 |
| Xe[10] | 1.3900 | 0.0110 | -0.670 | 3.46 | 4.75 | 4.50 | 3.90 | 0.359 | 0.370 | -0.154 | -0.156 |
| Kr[11] | 0.2190 | 0.0127 | -0.018 | 4.85 | 6.61 | 4.58 | 4.26 | 0.428 | 0.437 | 0.039 | 0.041 |
| $\operatorname{Ar}[10]$ | 0.2197 | 0.0306 | -0.458 | 3.82 | 5.21 | 4.37 | 4.22 | 0.549 | 0.510 | -0.180 | -0.175 |
| $\mathrm{Ne}[10]$ | 0.3740 | 0.0388 | -0.760 | 3.63 | 4.96 | 4.63 | 4.10 | 0.775 | 0.753 | -0.355 | -0.370 |
| $\mathrm{He}[10]$ | 0.3170 | 0.0106 | -0.010 | 4.43 | 6.03 | 4.04 | 3.75 | 1.190 | 1.200 | 0.123 | 0.127 |
| Zn line $307.6 \mathrm{~nm}, \mathrm{~T}=\mathbf{8 6 0} \mathrm{K}$ |  |  |  |  |  |  |  |  |  |  |  |
| $\mathrm{Zn}[12]$ | 1.7875 | 0.0199 | -0.475 | 3.67 | 5.02 | 4.22 | 4.07 | 0.491 | 0.441 | -0.163 | -0.161 |
| Xe[13] | 10.532 | 0.1926 | -2.558 | 3.98 | 5.46 | 9.36 | 7.71 | 1.220 | 1.188 | -0.665 | -0.705 |
| Kr[14] | 2.6630 | 0.0028 | -5.369 | 2.47 | 3.39 | 5.91 | 4.95 | 0.829 | 0.817 | -0.268 | -0.291 |
| Ar[14] | 0.1650 | 0.0072 | -0.012 | 4.61 | 6.31 | 4.25 | 3.95 | 0.514 | 0.515 | 0.051 | 0.053 |
| $\mathrm{Ne}[15]$ | 0.1337 | 0.0030 | -0.014 | 4.12 | 5.63 | 3.83 | 3.56 | 0.529 | 0.531 | 0.052 | 0.054 |

* the values of $\rho$ are in $\AA$ i units; $\Delta \mathrm{C}_{6}$ in units $10^{-32} \mathrm{~cm}^{6} \mathrm{rad} \mathrm{s}^{-1}$, and $\Delta \mathrm{C}_{12}$ in units $10^{-74} \mathrm{~cm}^{12} \mathrm{rad} \mathrm{s}^{-1}$, which are taken from the references given below:
[1] Bielski et al. (1985), [2] Dygdala (1988), [3] Dygdala et al. (1989), [4] Bielski et al. (1980), [5] Czuchaj et al. (1997), [6] Okunishi et al. (1990), [7] Kurosawa et al. (1998), [8] Petzold and Behmenburg (1978), [9] Helmi et al. (1996) , [10] Czuchaj and Stoll (1999), [11] Czajkowski et al. (1991), [12] Czajkowski and Kopersk (1999), [13] Wallace et al. (1991), [14] Wallace et al. (1992), [15] Koperski and Czajkowski (2000)


Fig. 2. The phase shift $\eta$ as a function of the impact parameter $\rho$, when $\eta_{\mathrm{E}} \leq-1.57$.


Fig. 3. The phase shift $\eta$ as a function of the impact parameter $\rho$, when $-1.57<\eta_{\mathrm{E}}<-0.63$.

## Conclusions

The following conclusions are based on the fact that the calculated coefficients $\beta_{c}$ and $\delta_{c}$ by simple analytical formulae obtained by the present authors, when the Lennard-Jones potential was applied, are in good agreement as shown in Table 1, with the corresponding coefficients obtained numerically by other authors. This has led to the following considerations:
(1) new impact parameters $\rho_{\mathrm{o}}, \rho_{\mathrm{E}}, \rho_{\mathrm{ob}}$, $\rho_{\mathrm{o} \mathrm{\delta} \delta}$ and $\rho_{\delta}$, which are firstly defined and obtained,
(2) the impact parameters $\rho_{o b}$, and $\rho_{o \delta}$ responsible, receptively, for the broadening and shift of spectral lines are different, and depend strongly on the values of $\Delta \mathrm{C}_{6}$ and $\Delta \mathrm{C}_{12}$, the values and signs of the phase shifts $\eta_{\mathrm{ob}}$ and $\eta_{\mathrm{o} \mathrm{\delta}}$ due to the broadening and shift,
(3) the phases $\eta_{\mathrm{ob}}$ and $\eta_{o \delta}$ at which the broadening and shift, respectively, start to take place are also different in values, which are the same for all interactions, $\eta_{\mathrm{ob}}= \pm 0.63$ and $\eta_{o \delta}= \pm 1.57$,
(4) the sign of phases $\eta_{o b}$ or $\eta_{o \delta}$ depended on the value of the phase $\eta_{\mathrm{E}}$ at the equilibrium position of phases, which value is given by (14),
(5) the impact parameter $\rho_{\delta}$ which separates between the positive and negative signs of the shift of spectral line was obtained and given by a simple formula (15), so that if $\rho_{\text {o }}$ $<\rho_{\delta}$, then the line was shifted to the blue wing ( $\delta=$ positive value), but if $\rho_{o \delta}>\rho_{\delta}$, then it was shifted to the red wing ( $\delta=$ negative value),
(6) as the calculated coefficients $\beta_{c}$ and $\delta_{c}$, using the approximated formulae, are in good agreement with those obtained before by numerical calculations, then all assumptions leading to the approximated formulae are valid and the obtained formulae, furthermore, can be easily used with other complicated potentials.

## References

Bielski, A., Bobkowski, R., Dokurno, W., Szudy, J. 1980. Low pressure shift of the $2 p^{5} 3 p-2 p^{5} 4 d$ spectral lines of neon perturbed by neon and helium. Acta Phys. Polonika A59: 107-114.
Bielski, A., Wawrzynski, J., Wolnikowski, J. 1985. Low pressure broadening and shift of the $3 \mathrm{P}^{5} 4 \mathrm{P}-3 \mathrm{P}^{5} 6 \mathrm{~s}$ argon spectral lines perturbed by halium, neon and argon. Acta Phys. Polonica A67: 621-629.
Czajkowski, M., Bobkowski, R., Krause, L. 1991. Spectroscopy of $\mathrm{AO}^{+} \leftarrow \mathrm{XO}^{+}$and $\mathrm{B} 1 \leftarrow \mathrm{XO}^{+}$transitions in cadmium krypton (Cd Kr). Phys. Rev. A 44: 5730-5736.
Czajkowski, M., Kopersk, J. J. 1999. The $\mathrm{Cd}_{2}$ and $\mathrm{Zn}_{2}$ van der Waals dimers revisited. Correction for some molecular potential parameters. Spectrochimica Acta A55: 2221-2229.
Czuchaj, E., Rebentrost, F., Stoll, H., Preuss, H. 1997. Calculation of the ground- and excited-state potential energy curves for the $\mathrm{Hg}_{2}$ molecule in a pseudopotential approach. Chem. Phys. 214: 277-289.
Czuchaj, E., Sienkiewicz, J. 1984. Pseudopotential calculation of the adiabatic potentials and oscillator strengths of cad-mium-rare gas pairs. J. Phys. B17: 2251-2267.
Czuchaj, E., Stoll, H. 1999. Calculation of ground and excitedstate potential energy curves for the Cd-rare gas complexes. Chem. Phys. 248: 1-16.
Dygdala, R. S. 1988. Broadening and shift of the resonance line of thallium due to collisions with krypton and xenon atoms. J. Phys. B:At. Mol. Opt. Phys. 21: 2039-2047.
Dygdala, R. S., Bobkowski, R., Lisicki, E. 1989. Broadening
and shift of the resonance line of thallium due to collisions with neon and argon atoms. J. Phys. B:At. Mol. Opt. Phys. 22: 1563-1572.
Findeisen, M., Grycuk, T., Bielski, A., Szudy, J. 1987. On the role of Maxwellian overaging in the interpretation of the pressure broadening and shift of spectral lines. J. Phys. B:At. Mol. Opt. Phys. 20: 5997-6003.
Helmi, M. S. 1994. On the validity of Weisskopf phase shift in the pressure broadening and shift of spectral lines. Pak. J. Sci. Ind. Res. 37: 499-502.

Helmi, M. S., Grycuk, T., Roston, G. D. 1996. Interatomic potentials and the van der Waals' coeffiicient for the intercombination Cd 326.6 nm absorption line broadened by cadmium pressure. Spectrochimica Acta B 51: 633-641.
Helmi, M. S., Roston, G. D. 2000. Determination of the phase shifts leading to the broadening and shift of spectral lines. Physica Scripta 62: 36-40.
Hindmarsh, W. R., Petford, A. D., Smith, G. 1967. Interpretation of collision broadening and shift of spectral lines. Proc. Roy. Soc. A297: 296-304.
Koperski, J., Czajkowski, M. 2000. Spectroscopic characteriza-
tion of the ZnNe van der Waals molecule in the $\mathrm{XO}^{+}\left(4^{1} \mathrm{~S}_{0}\right)$ and D1 ( $4^{1} \mathrm{P}_{1}$ ) energy states. Phys. Rev. A: At. Mol. Opt. Phys. 62: 012505/1-012505/9.
Kurosawa, T., Ohmori, K., Chiba, H., Okunishi, M., Ueda, K., Sato, Y. , Devdariani, A. Z., Nikitin, E. E. 1998. Collision induced absorption in mercury-rare gas collisions. J. Chem. Phys. 108: 8101-8109.
Okunishi, M., Nakazawa, H., Yamanouchi, K.,Tsuchiya, S. 1990. Interatomic potentials of mercury-krypton and mercuryxenon ( HgKr and HgXe ) van der Waals complexes in the $\mathrm{A}^{3} \mathrm{O}^{+}$and $\mathrm{B}^{3} 1$ states. Revisited. J. Chem. Phys. 93: 75267527.

Petzold, H. C., Behmenburg, W. 1978. Interaction potentials of mercury- argon from temperature dependent absorption spectra. Z. Naturforsch. 33A: 1461-1468.
Wallace, I., Kaup, J. G., Breckenridge, W. H. 1991. Excitation of $\mathrm{Zn}\left(4 s 4 \mathrm{P}^{\prime} \mathrm{P}_{1}\right)$ within the zinc - xenon van der Waals complex: effect of $p$-orbital alignment on the production of zinc ( $4 \mathrm{~s} 4 \mathrm{p}^{3} \mathrm{P}_{\mathrm{j}}$ ). J. Phys. Chem. 95: 8060-8065.
Wallace, I., Ryter, J., Breckenridge, W. H. 1992. Spectroscopic characterization of the $X^{1} \Sigma^{+}$and $C^{1} \Pi_{1}$ states of the zinckrypton (Zn-Kr) molecule. J. Chem. Phys. 96: 136-143.


[^0]:    *Author for correspondence; E-mail: gamal_daniel@yahoo.com

